REMARK ON WELL-POSEDNESS OF QUADRATIC SCHRÖDINGER EQUATION WITH NONLINEARITY $u\overline{u}$ IN $H^{-1/4}(\mathbb{R})$

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Abstract. In this note, we give another approach to the local well-posedness of quadratic Schrödinger equation with nonlinearity $u\overline{u}$ in $H^{-1/4}$, which was already proved by Kishimoto [3]. Our resolution space is l^1-analogue of $X^{s,b}$ space with low frequency part in a weaker space $L_t^\infty L_x^2$. Such type spaces were developed by Guo. [2] to deal the KdV endpoint $H^{-3/4}$ regularity.

1. Introduction

This paper is mainly concerned with the following equation

$$\begin{cases} iu_t + u_{xx} = |u|^2, \quad u(x,t) : \mathbb{R}\times\mathbb{R}\to\mathbb{C}, \\ u(x,0) = \phi(x) \in H^s(\mathbb{R}). \end{cases} \tag{1.1}$$

The low regularity for this equation was first studied by Kenig, Ponce, Vega in [4], they proved the local well-posedness in H^s, for $s > -1/4$, by using $X^{s,b}$ spaces. The local well-posedness in $H^{-1/4}$ was already proved by Kishimoto [3], where Kishimoto solved (1.1) in the spaces

$$Z = X^{-1/4,1/2+\beta} + Y$$

and

$$Y = \{f \in \mathcal{S}'(\mathbb{R}^2); \|f\|_Y = \|\langle\xi\rangle^{-1/4}\langle\tau-\xi^2\rangle^{3\beta}\hat{f}\|_{L_\xi^2 L_\tau^p} + \|\langle\xi\rangle^{1/4-2\beta}\langle\tau-\xi^2\rangle^{3\beta}\hat{f}\|_{L_\xi^2 L_\tau^2}\},$$

with $0 < \beta \leq 1/24$, $2\beta < 1/p' < 3\beta$ and $1/p + 1/p' = 1$.

We give another approach based on the argument developed by Guo. [2], which solved the global well-posedness for KdV equation in $H^{-3/4}$. Our resolution space is l^1-analogue of $X^{s,b}$ space with low frequency part in a weaker space $L_t^\infty L_x^2$, so as a resolution space, it has simple form.

It is well known that $X^{s,b}$ failed for (1.1) in $H^{-1/4}$ because of the logarithmic divergences from *high* $\times$ *high* $\to$ *low* interactions, it is natural to use the weaker structure in low frequency. We use $L_t^\infty L_x^2$ to measure the low frequency part, however in [2] Guo used $L_x^2 L_t^\infty$. The reason for this is that in the KdV case, the *high* $\times$ *low* interactions has one derivative, and the smoothing effect norm $L_x^\infty L_t^2$ was needed to absorb it. This method can also be adapted to other similar problems where some logarithmic divergences appear in the high-high interactions.

Theorem 1.1. *The initial value problem* (1.1) *is local well-posedness in* $H^{-1/4}$.

For $f \in \mathcal{S}'$ we denote by $\widehat{f}$ or $\mathcal{F}(f)$ the Fourier transform of f. We denote by $\mathcal{F}_x$ the Fourier transform on spatial variable. Let $\mathbb{Z}$ and $\mathbb{N}$ be the sets of integers and natural numbers respectively, $\mathbb{Z}_+ = \mathbb{N}\cup\{0\}$. For $k \in \mathbb{Z}_+$ let $I_k = \{\xi : |\xi| \in [2^{k-1}, 2^{k+1}]\}$, $k \geq 1$; $I_0 = \{\xi : |\xi| \leq 2\}$. Let $\eta_0 : \mathbb{R} \to [0,1]$ denote an even smooth function supported in $[-8/5, 8/5]$ and equal to 1 in $[-5/4, 5/4]$. We define $\psi(t) = \eta_0(t)$. For $k \in \mathbb{Z}$ let $\eta_k(\xi) = \eta_0(\xi/2^k) - \eta_0(\xi/2^{k-1})$ if $k \geq 1$ and $\eta_k(\xi) \equiv 0$ if $k \leq -1$. For $k \in \mathbb{Z}_+$, define P_k by $\widehat{P_k u}(\xi) = \eta_k(\xi)\widehat{u}(\xi)$. For $l \in \mathbb{Z}$ let $P_{\leq l} = \sum_{k\leq l} P_k$, $P_{\geq l} = \sum_{k\geq l} P_k$.

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For $u_0 \in \mathcal{S}'(\mathbb{R})$, we denote $W(t)u_0 = e^{it\partial_x^2}u_0$ defined by $\mathcal{F}_x(W(t)\phi)(\xi) = \exp[-i\xi^2 t]\widehat{\phi}(\xi)$.

For $k \in \mathbb{Z}_+$ we define the dyadic $X^{s,b}$-type normed spaces $X_k = X_k(\mathbb{R}^2)$,

$$X_k = \left\{ f \in L^2(\mathbb{R}^2) : \begin{array}{l} f(\xi,\tau) \text{ is supported in } I_k \times \mathbb{R} \text{ and} \\ \|f\|_{X_k} = \sum_{j=0}^{\infty} 2^{j/2} \|\eta_j(\tau+\xi^2)\cdot f\|_{L^2}, \end{array} \right\} \tag{1.2}$$

thus we have $\|\widehat{f}\|_{L^1_\tau L^2_\xi} \leq \|f\|_{X_k}$. For $-3/4 \leq s \leq 0$, we define our resolution spaces

$$\bar{F}^s = \{u \in \mathcal{S}'(\mathbb{R}^2) : \|u\|_{\bar{F}^s}^2 = \sum_{k\geq 1} 2^{2sk} \|\eta_k(\xi)\mathcal{F}(u)\|_{X_k}^2 + \|P_{\leq 0}(u)\|^2_{L_t^\infty L_x^2} < \infty\}. \tag{1.3}$$

It is easy to see that for $k \in \mathbb{Z}_+$

$$\|P_k(u)\|_{L_t^\infty L_x^2} \lesssim \|\mathcal{F}[P_k(u)]\|_{X_k}, \tag{1.4}$$

as a consequence, we have $\|u\|_{L_t^\infty H^s} \lesssim \|u\|_{\bar{F}^s}$.

Let $a_1, a_2, a_3 \in \mathbb{R}$, define $a_{max} = \max\{a_1, a_2, a_2\}$, same as a_{min}, a_{med}. Usually we use k_1, k_2, k_3 and j_1, j_2, j_3 to denote integers, $N_i = 2^{k_i}$ and $L_i = 2^{j_i}$ for $i = 1, 2, 3$ to denote dyadic numbers.

2. Dyadic Bilinear Estimates

In this section we will give some dyadic bilinear estimates for next section. We define

$$D_{k,j} = \{(\xi,\tau) : \xi \in [2^{k-1}, 2^{k+1}] \text{ and } \tau + \xi^2 \in I_j\}, \quad k \in \mathbb{Z}, j \in \mathbb{Z}_+.$$

Following the $[k; Z]$ methods [5] the bilinear estimates in $X^{s,b}$ space reduce to some dyadic summations: for any $k_1, k_2, k_3 \in \mathbb{Z}$ and $j_1, j_2, j_3 \in \mathbb{Z}_+$

$$\sup_{(u_{k_2,j_2},\ v_{k_3,j_3}) \in E} \|1_{D_{k_1,j_1}}(\xi,\tau) \cdot u_{k_2,j_2} * v_{k_3,j_3}(\xi,\tau)\|_{L^2_{\xi,\tau}} \tag{2.1}$$

where $E = \{(u,v) : \|u\|_2,\ \|v\|_2 \leq 1 \text{ and } \operatorname{supp}(u) \subset D_{k_2,j_2},\ \operatorname{supp}(v) \subset \widetilde{D}_{k_3,j_3}\}$ and $\widetilde{D}_{k_3,j_3} = \{(\xi,\tau); (-\xi,-\tau) \in D_{k_3,j_3}\}$. By checking the support properties, we get that in order for (2.1) to be nonzero one must have

$$|k_{max} - k_{med}| \leq 3, \text{ and } j_{max} \geq k_{max} + k_{min} - 10 \tag{2.2}$$

The following sharp estimates on (2.1) were obtained in [5].

Lemma 2.1 (Proposition 11.1, [5] (−++) case)**.** *Let $k_1, k_2, k_3 \in \mathbb{Z}$ and $j_1, j_2, j_3 \in \mathbb{Z}_+$. Let $N_i = 2^{k_i}$ and $L_i = 2^{j_i}$ for $i = 1, 2, 3$. Then*

(i) If $N_{max} \sim N_{min}$ and $L_{max} \sim N_{max}N_{min}$, then we have

$$(2.1) \lesssim L_{min}^{1/2} L_{med}^{1/4}. \tag{2.3}$$

(ii) If $N_1 \sim N_3 \gg N_2$ and $N_{max}N_{min} \sim L_2 = L_{max}$, and $N_1 \sim N_2 \gg N_3$ and $N_{max}N_{min} \sim L_3 = L_{max}$, then

$$(2.1) \lesssim L_{min}^{1/2} L_{med}^{1/2} N_{min}^{-1/2}. \tag{2.4}$$

(iii) In all other cases, we have

$$(2.1) \lesssim L_{min}^{1/2} N_{max}^{-1/2} \min(N_{max}N_{min}, L_{med})^{1/2}. \tag{2.5}$$

3. Proof of Theorem 1.1

For $u, v \in \bar{F}^s$ we define the bilinear operator

$$B(u,v) = \psi(\frac{t}{4}) \int_0^t W(t-\tau)\partial_x\big(\psi^2(\tau)u(\tau)\cdot v(\tau)\big)d\tau. \tag{3.1}$$

As in [2], the proof for Theorem 1.1 reduce to showing the boundness of $B : \bar{F}^{-1/4}\times\bar{F}^{-1/4} \to \bar{F}^{-1/4}$.

Lemma 3.1 (Linear estimates)**.** *(a) Assume $s \in \mathbb{R}$, $\phi \in H^s$. Then there exists $C > 0$ such that*

$$\|\psi(t)W(t)\phi\|_{\bar{F}^s} \le C\|\phi\|_{H^s}. \tag{3.2}$$

(b) Assume $s \in \mathbb{R}, k \in \mathbb{Z}_+$ and $(i+\tau-\xi^3)^{-1}\mathcal{F}(u) \in X_k$. Then there exists $C > 0$ such that

$$\left\|\mathcal{F}\left[\psi(t)\int_0^t W(t-s)(u(s))ds\right]\right\|_{X_k} \le C\|(i+\tau-\xi^3)^{-1}\mathcal{F}(u)\|_{X_k}. \tag{3.3}$$

Proof. Such linear estimates have appeared in many literatures, see for example [1]. □

Lemma 3.2 (Bilinear estimates)**.** *Assume $-1/4 \le s \le 0$. Then there exists $C > 0$ such that*

$$\|B(u,v)\|_{\bar{F}^s} \le C(\|u\|_{\bar{F}^s}\|v\|_{\bar{F}^{-1/4}} + \|u\|_{\bar{F}^{-1/4}}\|v\|_{\bar{F}^s}) \tag{3.4}$$

hold for any $u, v \in \bar{F}^s$.

Proof. It is easy to see

$$\begin{aligned}\|B(u,v)\|_{\bar{F}^s} \lesssim &\|P_{\ge1}B(P_{\ge1}u, P_{\ge1}v)\|_{F^s} + \|P_{\ge1}B(P_{\ge1}u, P_0v)\|_{\bar{F}^s}\\ &+ \|P_{\ge1}B(P_0u, P_{\ge1}v)\|_{\bar{F}^s} + \|P_{\ge1}B(P_0u, P_0v)\|_{\bar{F}^s} + \|P_0B(u,v)\|_{\bar{F}^s}\\ \triangleq &A + B + C + D + E\end{aligned}$$

We notice that there is no low frequency in part A, so the proof for part A do not involve the special structure in low frequency, and standard $X^{s,b}$ argument will suffice, we omit the proof.

The proof for part B, C and D are similar, we just consider part B for example. By definition and Lemma 3.1 (b), let $S_B = \{(k_1,k_3); k_1,k_3 \ge 1, |k_1-k_3| \le 5\}$, then

$$\begin{aligned}B^2 \lesssim &\sum_{(k_1,k_3)\in S_B} 2^{2sk_3}\Big(\sum_{j_3\ge0} 2^{-j_3/2}\|1_{D_{k_3,j_3}}\widehat{\psi(t)P_{k_1}}u * \widehat{P_0v}\|_{L^2_{\xi,\tau}}\Big)^2\\ \lesssim &\sum_{(k_1,k_3)\in S_B} 2^{2sk_3}\|\psi(t)P_{k_1}u\|^2_{L^2}\|P_0v\|^2_{L^\infty} \lesssim \sum_{(k_1,k_3)\in S_B} 2^{2sk_3}\|P_{k_1}u\|^2_{L^\infty_tL^2_x}\|P_0v\|^2_{L^\infty}\end{aligned} \tag{3.5}$$

which is sufficient by Bernstein inequality and (1.4).

Now we turn to part D. Denote $Q(u,v) = P_{\le0}B(P_{k_1}u, P_{k_2}\bar{v})$. By straightforward computations,

$$\mathcal{F}\left[Q(u,\bar{v})\right](\xi,\tau) = c\int_{\mathbb{R}} \frac{\widehat{\psi}(\tau-\tau') - \widehat{\psi}(\tau+\xi^2)}{\tau'+\xi^2}\eta_0(\xi)\int_Z \widehat{P_{k_1}u}(\xi_1,\tau_1)\widehat{P_{k_2}\bar{v}}(\xi_2,\tau_2)\,d\tau'.$$

where $Z = \{\xi = \xi_1+\xi_2, \tau' = \tau_1+\tau_2\}$. Fixing $\xi \in \mathbb{R}$, we decomposing the hyperplane as following

$$\begin{aligned}A_1 &= \{\xi = \xi_1+\xi_2, \tau' = \tau_1+\tau_2 : |\xi| \lesssim 2^{-k_1}\};\\ A_2 &= \{\xi = \xi_1+\xi_2, \tau' = \tau_1+\tau_2 : |\xi| \gg 2^{-k_1},\\ &\quad |\tau_1+\xi_1^2| \ll 2^{k_1}|\xi|, |\tau_2-\xi_2^2| \ll 2^{k_1}|\xi|\};\\ A_3 &= \{\xi = \xi_1+\xi_2, \tau' = \tau_1+\tau_2 : |\xi| \gg 2^{-k_1}, |\tau_1+\xi_1^2| \gtrsim 2^{k_1}|\xi|\};\\ A_4 &= \{\xi = \xi_1+\xi_2, \tau' = \tau_1+\tau_2 : |\xi| \gg 2^{-k_1}, |\tau_2-\xi_2^2| \gtrsim 2^{k_1}|\xi|\}.\end{aligned}$$

Then we get

$$\mathcal{F}\left[Q(u,\bar{v})\right](\xi,\tau) = I + II + III,$$

where

$$\begin{aligned}
I =& C\int_{\mathbb{R}} \frac{\widehat{\psi}(\tau-\tau')-\widehat{\psi}(\tau+\xi^2)}{\tau'+\xi^2}\eta_0(\xi)\int_{A_1}\widehat{P_{k_1}u}(\xi_1,\tau_1)\widehat{P_{k_2}\bar{v}}(\xi_2,\tau_2)d\tau',\\
II =& C\int_{\mathbb{R}} \frac{\widehat{\psi}(\tau-\tau')-\widehat{\psi}(\tau+\xi^2)}{\tau'+\xi^2}\eta_0(\xi)\int_{A_2}\widehat{P_{k_1}u}(\xi_1,\tau_1)\widehat{P_{k_2}\bar{v}}(\xi_2,\tau_2)d\tau',\\
III =& C\int_{\mathbb{R}} \frac{\widehat{\psi}(\tau-\tau')-\widehat{\psi}(\tau+\xi^2)}{\tau'+\xi^2}\eta_0(\xi)\int_{A_3\cup A_4}\widehat{P_{k_1}u}(\xi_1,\tau_1)\widehat{P_{k_2}\bar{v}}(\xi_2,\tau_2)d\tau'.
\end{aligned}$$

We consider first the the term I. By (1.4) and Proposition 3.1 (b),

$$\|\mathcal{F}^{-1}(I)\|_{L_t^\infty L_x^2}\lesssim\|I\|_{X_0}\lesssim\left\|(i+\tau'+\xi^2)^{-1}\eta_0(\xi)\int_{A_1}\widehat{P_{k_1}u}(\xi_1,\tau_1)\widehat{P_{k_2}\bar{v}}(\xi_2,\tau_2)\right\|_{X_0},$$

since in the set A_1 we have $|\xi|\lesssim 2^{-k_1}$, thus we continue with

$$\lesssim \sum_{k_3\leq -k_1+10}\sum_{j_3\geq 0}2^{-j_3/2}\sum_{j_1\geq 0, j_2\geq 0}\|1_{D_{k_3,j_3}}\cdot u_{k_1,j_1} * v_{k_2,j_2}\|_{L^2}$$

where

$$u_{k_1,j_1} = \eta_{k_1}(\xi)\eta_{j_1}(\tau+\xi^2)\widehat{u},\ v_{k,j_2} = \eta_k(\xi)\eta_{j_2}(\tau-\xi^2)\widehat{\bar{v}}. \tag{3.6}$$

Using Proposition 2.1 (iii), then we get

$$\begin{aligned}
\|\mathcal{F}^{-1}(I)\|_{L_t^\infty L_x^2} \;&\lesssim\; \sum_{k_3\leq -k_1+10}\sum_{j_i\geq 0}2^{-j_3/2}2^{j_{min}/2}2^{k_3/2}\|u_{k_1,j_1}\|_{L^2}\|v_{k_2,j_2}\|_{L^2}\\
&\lesssim\; 2^{-k_1/2}\|\widehat{P_{k_1}u}\|_{X_{k_1}}\|\widehat{P_{k_2}v}\|_{X_{k_2}},
\end{aligned}$$

which suffices to give the bound for the term I since $|k_1-k_2|\leq 5$.

Next we consider the contribution of the term III. As term I, by (1.4) and Proposition 3.1 (b),

$$\begin{aligned}
\|\mathcal{F}^{-1}(III)\|_{L_t^\infty L_x^2} \;&\lesssim\; \left\|(i+\tau'+\xi^2)^{-1}\eta_0(\xi)\int_{A_3\cup A_4}\widehat{P_{k_1}u}(\xi_1,\tau_1)\widehat{P_{k_2}\bar{v}}(\xi_2,\tau_2)\right\|_{X_0}\\
&\lesssim\; \sum_{-k_1\leq k_3\leq 0}\sum_{j_3\geq 0}2^{-j_3/2}\sum_{j_1\geq 0, j_2\geq 0}\|1_{D_{k_3,j_3}}\cdot u_{k_1,j_1} * v_{k_2,j_2}\|_{L^2}.
\end{aligned}$$

Without loss of generality, we assume $|\tau_1+\xi_1^2|\gtrsim|\xi\xi_1|$, applying Proposition 2.1 (iii), then we get

$$\begin{aligned}
\|\mathcal{F}^{-1}(III)\|_{L_t^\infty L_x^2} \;&\lesssim\; \sum_{-k_1\leq k_3\leq 0}\ \sum_{j_1\geq k_3+k_1-10, j_2\geq 0}2^{j_2/2}2^{-k_1/2}\|u_{k_1,j_1}\|_{L^2}\|v_{k_2,j_2}\|_{L^2}\\
&\lesssim\; 2^{-k_1/2}\|\widehat{P_{k_1}u}\|_{X_{k_1}}\|\widehat{P_{k_2}u}\|_{X_{k_2}},
\end{aligned}$$

which suffices to give the bound for the term III since $|k_1-k_2|\leq 5$.

Now we consider the main contribution term: term II. By direct computation, we get

$$\mathcal{F}_t^{-1}(II) = \psi(t)\int_0^t e^{-i(t-s)\xi^2}\eta_0(\xi)i\xi\int_{\mathbb{R}^2}e^{is(\tau_1+\tau_2)}\int_{\xi=\xi_1+\xi_2}u_{k_1}(\xi_1,\tau_1)v_{k_2}(\xi_2,\tau_2)\ d\tau_1 d\tau_2 ds$$

where

$$u_{k_1}(\xi_1,\tau_1) = \eta_{k_1}(\xi_1)1_{\{|\tau_1+\xi_1^2|\ll 2^{k_1}|\xi|\}}\widehat{u}(\xi_1,\tau_1),\ v_{k_2}(\xi_2,\tau_2) = \eta_{k_2}(\xi_2)1_{\{|\tau_2-\xi_2^2|\ll 2^{k_1}|\xi|\}}\widehat{\bar{v}}(\xi_2,\tau_2).$$

By a change of variable $\tau_1' = \tau_1 + \xi_1^2$, $\tau_2' = \tau_2 - \xi_2^2$, we get

$$\begin{aligned}
\mathcal{F}_t^{-1}(II) &= \psi(t)e^{-it\xi^2}\eta_0(\xi)\int_0^t e^{is\xi^2}\int_{\mathbb{R}^2} e^{is(\tau_1+\tau_2)} \\
&\quad \times \int_{\xi=\xi_1+\xi_2} e^{-is\xi_1^2} u_{k_1}(\xi_1, \tau_1 - \xi_1^2) e^{is\xi_2^2} v_{k_2}(\xi_2, \tau_2 + \xi_2^2)\ d\tau_1 d\tau_2 ds \\
&= \psi(t)e^{-it\xi^2}\eta_0(\xi)\int_{\mathbb{R}^2} e^{it(\tau_1+\tau_2)}\int_{\xi=\xi_1+\xi_2} \frac{e^{it(-\xi_1^2+\xi_2^2+\xi^2)} - e^{-it(\tau_1+\tau_2)}}{\tau_1+\tau_2-\xi_1^2+\xi_2^2+\xi^2} \\
&\quad \times u_{k_1}(\xi_1, \tau_1 - \xi_1^2) v_{k_2}(\xi_2, \tau_2 + \xi_2^2)\ d\tau_1 d\tau_2.
\end{aligned}$$

Then by Plancherel Theorem and Hölder inequality, we can bound $\|\mathcal{F}_t^{-1}(II)\|_{L_\xi}$ by

$$\begin{aligned}
&\int_{\mathbb{R}^2} \Big\| \int_{\xi=\xi_1+\xi_2} \frac{\eta_0(\xi)}{|\tau_1+\tau_2-\xi_1^2+\xi_2^2+\xi^2|} |u_{k_1}(\xi_1, \tau_1-\xi_1^2) v_{k_2}(\xi_2, \tau_2+\xi_2^2)| \Big\|_{L_\xi^2} d\tau_1 d\tau_2 \\
\lesssim &\int_{\mathbb{R}^2} \sum_{-k_1 \le k \le 0} 2^{k/2} \Big\| \int_{\xi=\xi_1+\xi_2} \frac{\chi_k(\xi)}{|\xi\xi_1|} |u_{k_1}(\xi_1, \tau_1-\xi_1^2) v_{k_2}(\xi_2, \tau_2+\xi_2^2)| \Big\|_{L_\xi^\infty} d\tau_1 d\tau_2 \\
\lesssim & 2^{-k_1/2} \|u_{k_1}\|_{L^1_{\tau_2} L^2_{\xi_2}} \|v_{k_2}\|_{L^1_{\tau_3} L^2_{\xi_3}} \lesssim 2^{-k_1/2} \|\widehat{P_{k_1} u}\|_{X_{k_1}} \|\widehat{P_{k_2} u}\|_{X_{k_2}}.
\end{aligned}$$

where we use $|\tau_1+\tau_2-\xi_1^2+\xi_2^2+\xi^2| \gtrsim |\xi\xi_1|$, which completes the proof of the lemma. □

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References

[1] A. D. Ionescu, C. E. Kenig, Global well-posedness of the Benjamin-Ono equation in low-regularity spaces, J. Amer. Math. Soc., 20 (2007), no. 3, 753-798.

[2] Z. Guo, Global well-posedness of Korteweg-de Vries equationin $H^{-1/4}(\mathbb{R})$. J.Math.Pures Appl.91(2009)583-597

[3] N. Kishimoto, Low-regularity local well-posedness for quadratic nonlinear Schrödinger equations, preprint.

[4] C. E. Kenig, G. Ponce, and L. Vega, Quadratic forms for the 1-D semilinear Schrödinger equation, Trans. Amer. Math. Soc. 348 (1996), no. 8, 3323-3353.

[5] T. Tao, Multiplinear weighted convolution of L^2 functions and applications to nonlinear dispersive equations. Amer. J. Math. (2001), 123(5):839-908, .

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